Axiom-based Probabilistic Description Logic

Martin Unold & Christophe Cruz

Outline

- Classical Probabilistic DL
- Motivation
- Axiom-based Probabilistic DL

Crisp KnowledgeBase

$$\mathcal{K} = (\mathcal{T}, \mathcal{A})$$

$$\mathcal{T} = \{\mathtt{C} \sqsubseteq \mathtt{D}\}$$

$$\begin{aligned}\mathcal{A} = \{&\mathtt{a} : \mathtt{C},\\ &\mathtt{a} : \mathtt{C} \sqcup \mathtt{D},\\ &\mathtt{a} : \neg\mathtt{D}\}\end{aligned}$$

Crisp KnowledgeBase Concepts

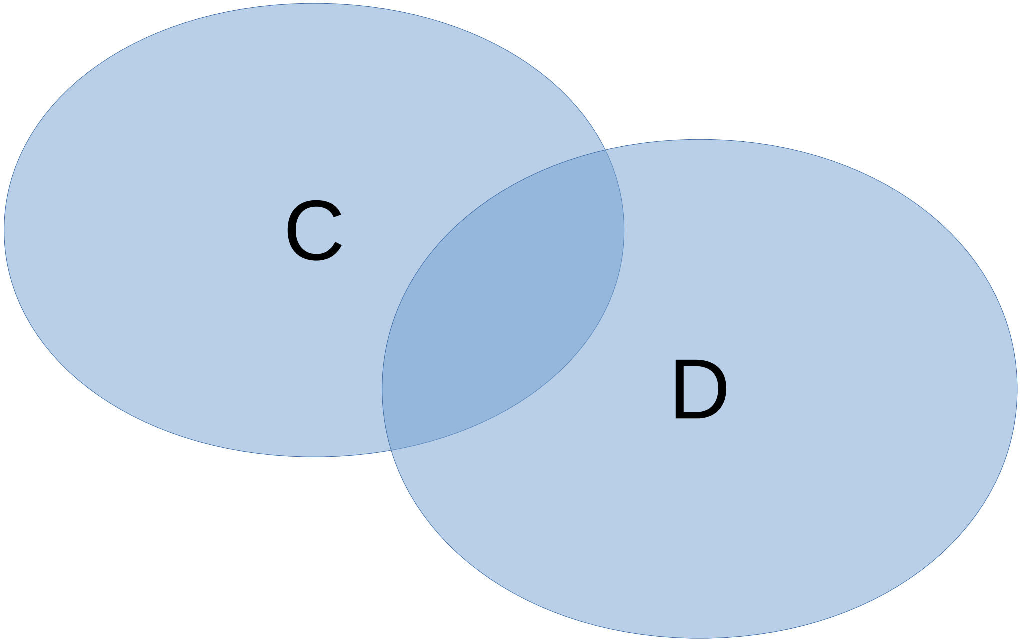
$$\mathcal{K} = (\mathcal{T}, \mathcal{A})$$

$$\mathcal{T} = \{\mathtt{C} \sqsubseteq \mathtt{D}\}$$

$$\mathcal{A} = \{\mathtt{a} : \mathtt{C},$$

$$\mathtt{a} : \mathtt{C} \sqcup \mathtt{D},$$

$$\mathtt{a} : \neg\mathtt{D}\}$$



Crisp KnowledgeBase Individuals

$$\mathcal{K} = (\mathcal{T}, \mathcal{A})$$

$$\mathcal{T} = \{\mathtt{C} \sqsubseteq \mathtt{D}\}$$

$$\begin{aligned} \mathcal{A} = \{&\mathtt{a} : \mathtt{C}, \\ &\mathtt{a} : \mathtt{C} \sqcup \mathtt{D}, \\ &\mathtt{a} : \neg \mathtt{D}\} \end{aligned}$$

Crisp KnowledgeBase

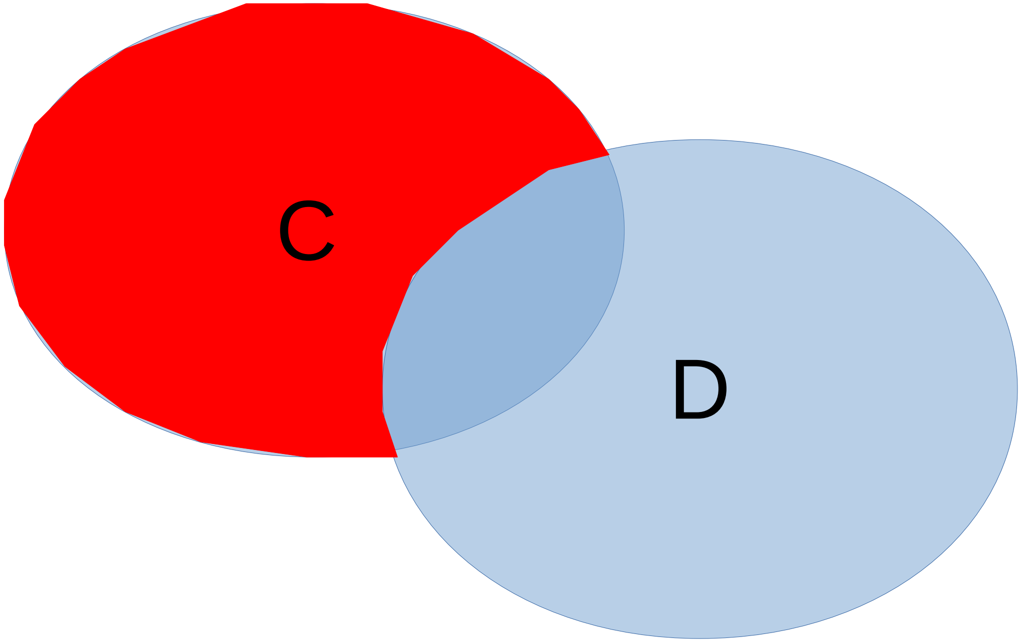
$$\mathcal{K} = (\mathcal{T}, \mathcal{A})$$

$$\mathcal{T} = \boxed{\{\mathtt{C} \sqsubseteq \mathtt{D}\}}$$

$$\mathcal{A} = \{\mathtt{a} : \mathtt{C},$$

$$\mathtt{a} : \mathtt{C} \sqcup \mathtt{D},$$

$$\mathtt{a} : \neg\mathtt{D}\}$$



Crisp KnowledgeBase

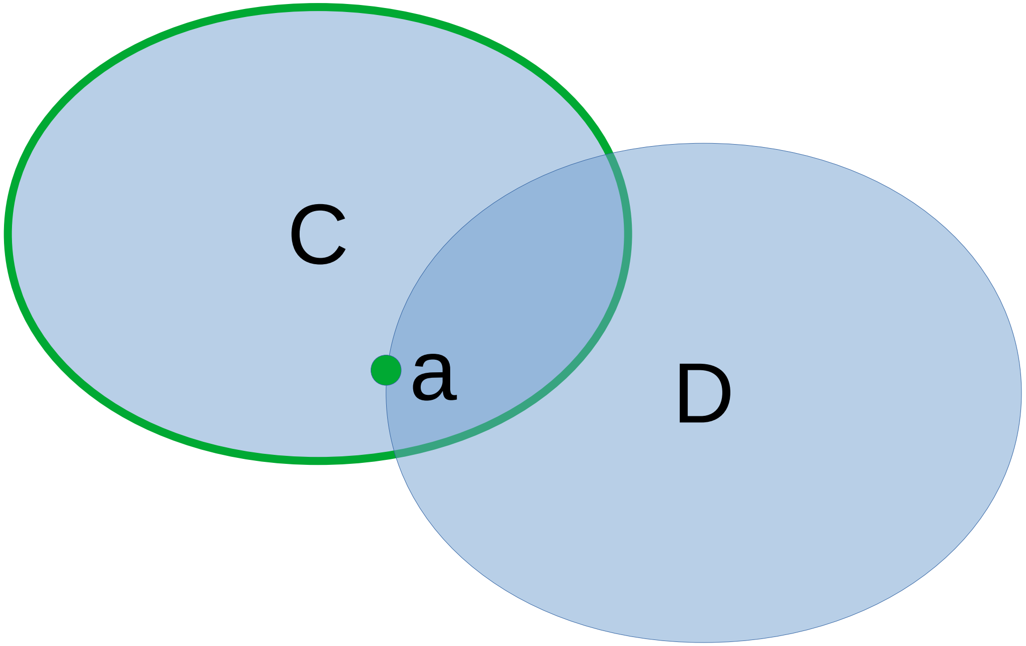
$$\mathcal{K} = (\mathcal{T}, \mathcal{A})$$

$$\mathcal{T} = \{\mathtt{C} \sqsubseteq \mathtt{D}\}$$

$$\mathcal{A} = \boxed{\{\mathtt{a} : \mathtt{C},}$$

$$\mathtt{a} : \mathtt{C} \sqcup \mathtt{D},$$

$$\mathtt{a} : \neg\mathtt{D}\}$$

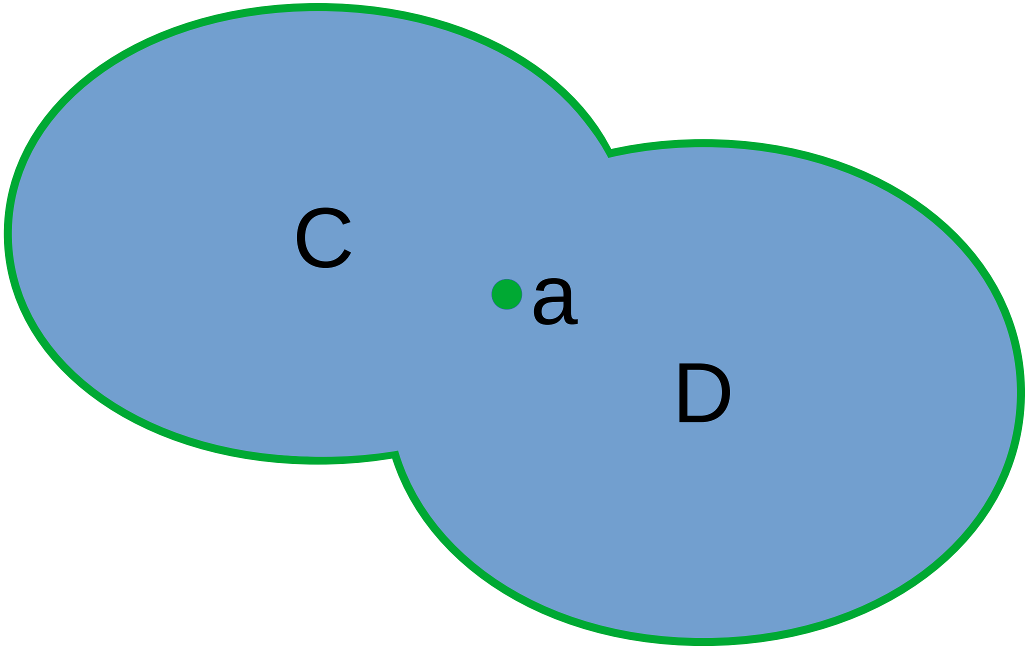


Crisp KnowledgeBase

$$\mathcal{K} = (\mathcal{T}, \mathcal{A})$$

$$\mathcal{T} = \{\mathtt{C} \sqsubseteq \mathtt{D}\}$$

$$\begin{aligned}\mathcal{A} = \{&\mathtt{a} : \mathtt{C},\\ &\boxed{\mathtt{a} : \mathtt{C} \sqcup \mathtt{D},}\\ &\mathtt{a} : \neg\mathtt{D}\}\end{aligned}$$

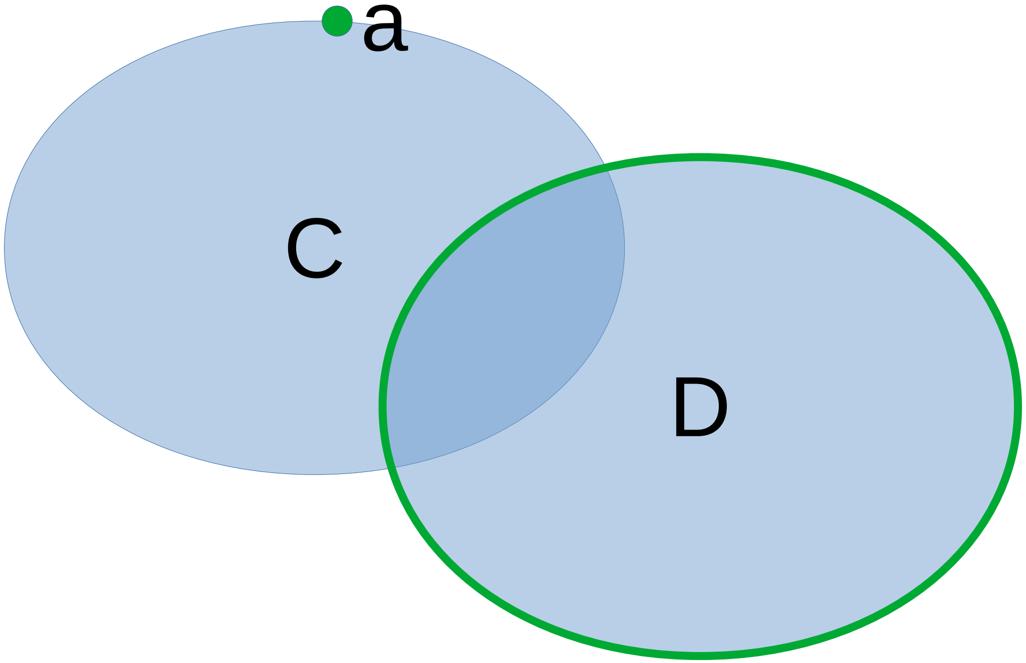


Crisp KnowledgeBase

$$\mathcal{K} = (\mathcal{T}, \mathcal{A})$$

$$\mathcal{T} = \{\mathtt{C} \sqsubseteq \mathtt{D}\}$$

$$\begin{aligned}\mathcal{A} = \{&\mathtt{a} : \mathtt{C}, \\ &\mathtt{a} : \mathtt{C} \sqcup \mathtt{D}, \\ &\boxed{\mathtt{a} : \neg \mathtt{D}\}}\end{aligned}$$



Possible Worlds

$$I_1 : \quad \mathtt{a}^{I_1} \in \mathtt{C}^{I_1}, \mathtt{a}^{I_1} \in \mathtt{D}^{I_1}$$

$$I_2 : \quad \mathtt{a}^{I_2} \in \mathtt{C}^{I_2}, \mathtt{a}^{I_2} \notin \mathtt{D}^{I_2}$$

$$I_3 : \quad \mathtt{a}^{I_3} \notin \mathtt{C}^{I_3}, \mathtt{a}^{I_3} \in \mathtt{D}^{I_3}$$

$$I_4 : \quad \mathtt{a}^{I_4} \notin \mathtt{C}^{I_4}, \mathtt{a}^{I_4} \notin \mathtt{D}^{I_4}$$

Possible Worlds

$I_1 : \quad \mathtt{a}^{I_1} \in \mathtt{C}^{I_1}, \mathtt{a}^{I_1} \in \mathtt{D}^{I_1}$

$I_2 : \quad \mathtt{a}^{I_2} \in \mathtt{C}^{I_2}, \mathtt{a}^{I_2} \notin \mathtt{D}^{I_2}$

$I_3 : \quad \mathtt{a}^{I_3} \notin \mathtt{C}^{I_3}, \mathtt{a}^{I_3} \in \mathtt{D}^{I_3}$

$I_4 : \quad \mathtt{a}^{I_4} \notin \mathtt{C}^{I_4}, \mathtt{a}^{I_4} \notin \mathtt{D}^{I_4}$

Possible Worlds

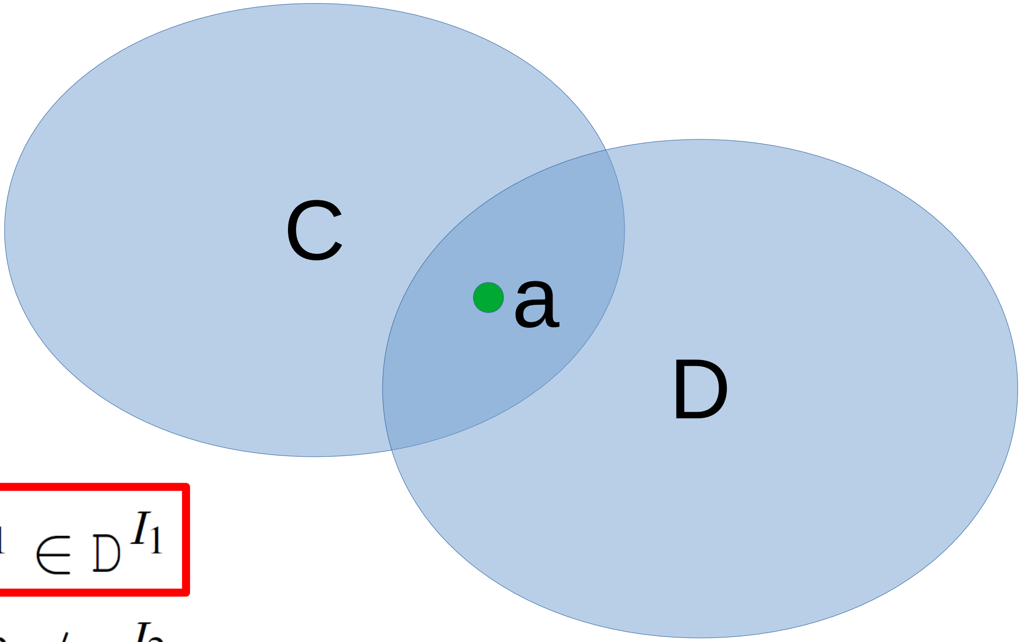
$\mathcal{K} = (\mathcal{T}, \mathcal{A})$

$\mathcal{T} = \{\mathtt{C} \sqsubseteq \mathtt{D}\}$

$\mathcal{A} = \{\mathtt{a} : \mathtt{C},$

$\mathtt{a} : \mathtt{C} \sqcup \mathtt{D},$

$\boxed{\mathtt{a} : \neg\mathtt{D}\}}$



$\boxed{I_1 : \quad \mathtt{a}^{I_1} \in \mathtt{C}^{I_1}, \mathtt{a}^{I_1} \in \mathtt{D}^{I_1}}$

$I_2 : \quad \mathtt{a}^{I_2} \in \mathtt{C}^{I_2}, \mathtt{a}^{I_2} \notin \mathtt{D}^{I_2}$

$I_3 : \quad \mathtt{a}^{I_3} \notin \mathtt{C}^{I_3}, \mathtt{a}^{I_3} \in \mathtt{D}^{I_3}$

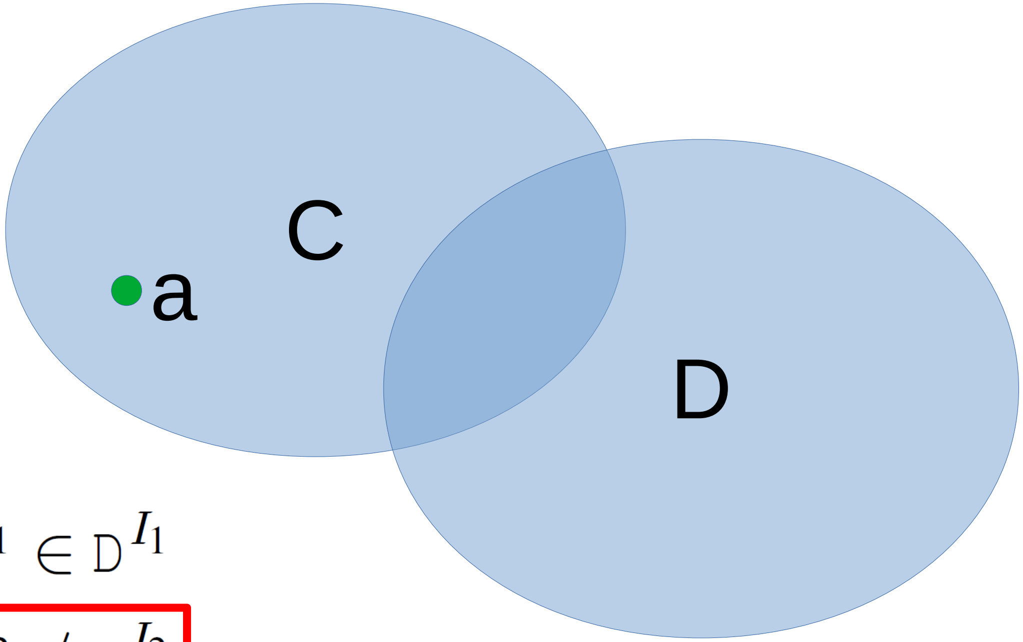
$I_4 : \quad \mathtt{a}^{I_4} \notin \mathtt{C}^{I_4}, \mathtt{a}^{I_4} \notin \mathtt{D}^{I_4}$

Possible Worlds

$$\mathcal{K} = (\mathcal{T}, \mathcal{A})$$

$$\mathcal{T} = \boxed{\{\mathtt{C} \sqsubseteq \mathtt{D}\}}$$

$$\begin{aligned}\mathcal{A} = \{&\mathtt{a} : \mathtt{C},\\ &\mathtt{a} : \mathtt{C} \sqcup \mathtt{D},\\ &\mathtt{a} : \neg\mathtt{D}\}\end{aligned}$$



$$I_1 : \quad \mathtt{a}^{I_1} \in \mathtt{C}^{I_1}, \mathtt{a}^{I_1} \in \mathtt{D}^{I_1}$$

$$\boxed{I_2 : \quad \mathtt{a}^{I_2} \in \mathtt{C}^{I_2}, \mathtt{a}^{I_2} \notin \mathtt{D}^{I_2}}$$

$$I_3 : \quad \mathtt{a}^{I_3} \notin \mathtt{C}^{I_3}, \mathtt{a}^{I_3} \in \mathtt{D}^{I_3}$$

$$I_4 : \quad \mathtt{a}^{I_4} \notin \mathtt{C}^{I_4}, \mathtt{a}^{I_4} \notin \mathtt{D}^{I_4}$$

Possible Worlds

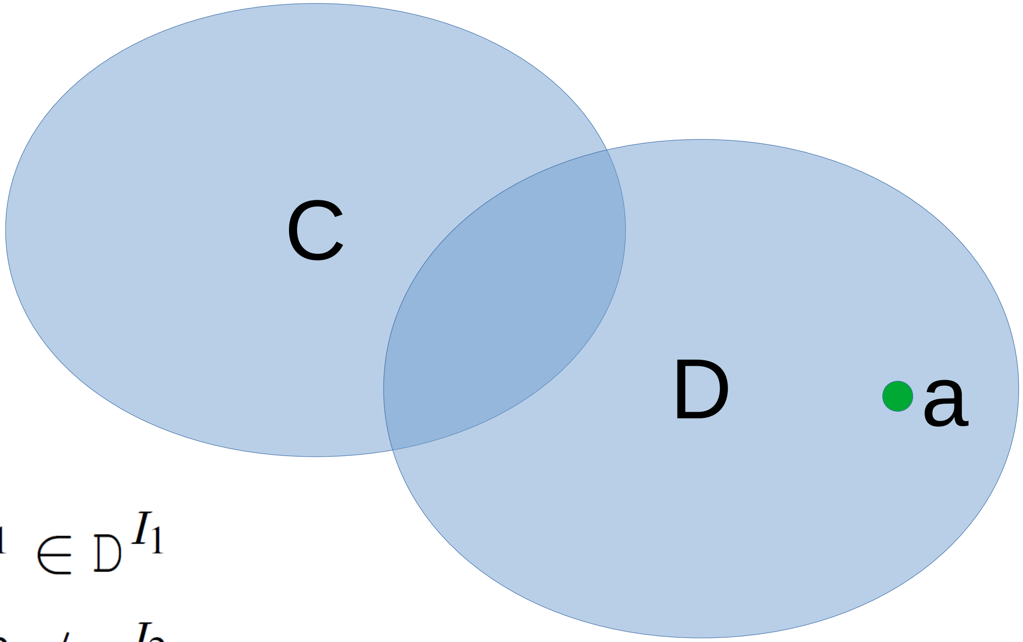
$\mathcal{K} = (\mathcal{T}, \mathcal{A})$

$\mathcal{T} = \{\mathtt{C} \sqsubseteq \mathtt{D}\}$

$\boxed{\mathcal{A} = \{\mathtt{a} : \mathtt{C},}$

$\mathtt{a} : \mathtt{C} \sqcup \mathtt{D},$

$\boxed{\mathtt{a} : \neg\mathtt{D}\}}$



$I_1 : \quad \mathtt{a}^{I_1} \in \mathtt{C}^{I_1}, \mathtt{a}^{I_1} \in \mathtt{D}^{I_1}$

$I_2 : \quad \mathtt{a}^{I_2} \in \mathtt{C}^{I_2}, \mathtt{a}^{I_2} \notin \mathtt{D}^{I_2}$

$\boxed{I_3 : \quad \mathtt{a}^{I_3} \notin \mathtt{C}^{I_3}, \mathtt{a}^{I_3} \in \mathtt{D}^{I_3}}$

$I_4 : \quad \mathtt{a}^{I_4} \notin \mathtt{C}^{I_4}, \mathtt{a}^{I_4} \notin \mathtt{D}^{I_4}$

Possible Worlds

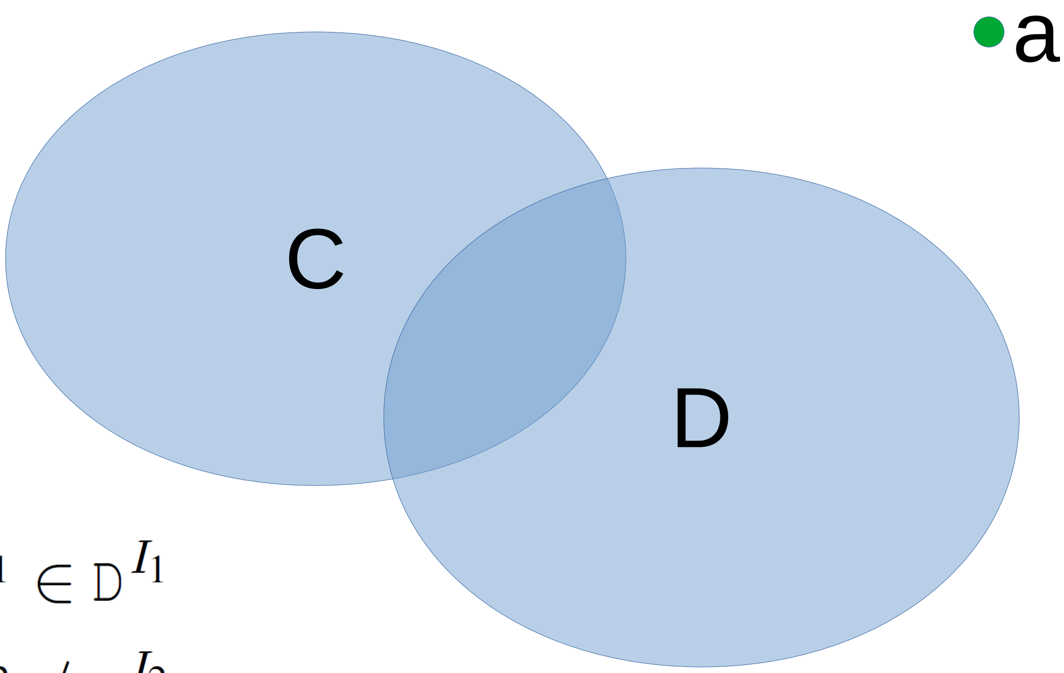
$$\mathcal{K} = (\mathcal{T}, \mathcal{A})$$

$$\mathcal{T} = \{\mathtt{C} \sqsubseteq \mathtt{D}\}$$

$$\mathcal{A} = \boxed{\{\mathtt{a} : \mathtt{C},}$$

$$\boxed{\mathtt{a} : \mathtt{C} \sqcup \mathtt{D},}$$

$$\mathtt{a} : \neg \mathtt{D}\}$$



$$I_1 : \quad \mathtt{a}^{I_1} \in \mathtt{C}^{I_1}, \mathtt{a}^{I_1} \in \mathtt{D}^{I_1}$$

$$I_2 : \quad \mathtt{a}^{I_2} \in \mathtt{C}^{I_2}, \mathtt{a}^{I_2} \notin \mathtt{D}^{I_2}$$

$$I_3 : \quad \mathtt{a}^{I_3} \notin \mathtt{C}^{I_3}, \mathtt{a}^{I_3} \in \mathtt{D}^{I_3}$$

$$\boxed{I_4 : \quad \mathtt{a}^{I_4} \notin \mathtt{C}^{I_4}, \mathtt{a}^{I_4} \notin \mathtt{D}^{I_4}}$$

Probabilistic KB

$$\mathcal{K} = (\mathcal{T}, \mathcal{A})$$

$$\mathcal{T} = \{0.75 :: \mathtt{C} \sqsubseteq \mathtt{D}\}$$

$$\begin{aligned}\mathcal{A} = \{&0.5 :: \mathtt{a} : \mathtt{C},\\ &0.5 :: \mathtt{a} : \mathtt{C} \sqcup \mathtt{D},\\ &0.75 :: \mathtt{a} : \neg\mathtt{D}\}\end{aligned}$$

Probabilistic KB

$$(p :: \phi) \in \mathcal{K}$$

$$\begin{bmatrix} a_{\phi 1} & a_{\phi 2} & a_{\phi 3} & a_{\phi 4} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \\ \pi_4 \end{bmatrix} = \begin{bmatrix} p \\ \vdots \\ 1 \end{bmatrix}$$

$$a_{\phi j} = \begin{cases} 1 & \text{if } \phi \models I_j \\ 0 & \text{if } \phi \not\models I_j \end{cases}$$

Probabilistic KB

$$\mathcal{K} = (\mathcal{T}, \mathcal{A})$$

$$\mathcal{T} = \{0.75 :: \mathtt{C} \sqsubseteq \mathtt{D}\}$$

$$\begin{aligned} \mathcal{A} = \{&0.5 :: \mathtt{a} : \mathtt{C}, \\ &0.5 :: \mathtt{a} : \mathtt{C} \sqcup \mathtt{D}, \\ &0.75 :: \mathtt{a} : \neg\mathtt{D}\} \end{aligned}$$

$$\begin{array}{cccc} \mathtt{a}^{I_1} \in \mathtt{C}^{I_1}, \mathtt{a}^{I_1} \in \mathtt{D}^{I_1} & \mathtt{a}^{I_2} \in \mathtt{C}^{I_2}, \mathtt{a}^{I_2} \notin \mathtt{D}^{I_2} & \mathtt{a}^{I_3} \notin \mathtt{C}^{I_3}, \mathtt{a}^{I_3} \in \mathtt{D}^{I_3} & \mathtt{a}^{I_4} \notin \mathtt{C}^{I_4}, \mathtt{a}^{I_4} \notin \mathtt{D}^{I_4} \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{array} \cdot \begin{bmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \\ \pi_4 \end{bmatrix} = \begin{bmatrix} 0.75 \\ 0.5 \\ 0.5 \\ 0.75 \\ 1 \end{bmatrix}$$

Probabilistic KB

$$\mathcal{K} = (\mathcal{T}, \mathcal{A})$$

$$\mathcal{T} = \{0.75 :: \mathtt{C} \sqsubseteq \mathtt{D}\}$$

$$\begin{aligned}\mathcal{A} = \{&0.5 :: \mathtt{a} : \mathtt{C},\\ &0.5 :: \mathtt{a} : \mathtt{C} \sqcup \mathtt{D},\\ &0.75 :: \mathtt{a} : \neg \mathtt{D}\}\end{aligned}$$

$$\begin{array}{cccc} \mathtt{a}^{I_1} \in \mathtt{C}^{I_1}, \mathtt{a}^{I_1} \in \mathtt{D}^{I_1} & \mathtt{a}^{I_2} \in \mathtt{C}^{I_2}, \mathtt{a}^{I_2} \notin \mathtt{D}^{I_2} & \mathtt{a}^{I_3} \notin \mathtt{C}^{I_3}, \mathtt{a}^{I_3} \in \mathtt{D}^{I_3} & \mathtt{a}^{I_4} \notin \mathtt{C}^{I_4}, \mathtt{a}^{I_4} \notin \mathtt{D}^{I_4} \\ \end{array}$$

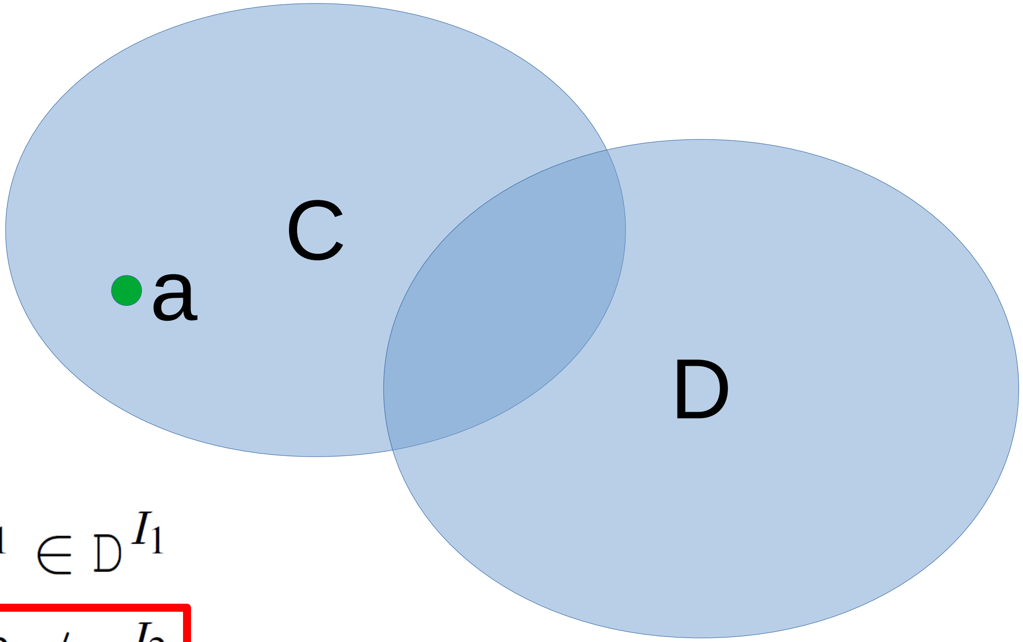
$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \\ \pi_4 \end{bmatrix} = \begin{bmatrix} 0.75 \\ 0.5 \\ 0.5 \\ 0.75 \\ 1 \end{bmatrix}$$

Probabilistic KB

$$\mathcal{K} = (\mathcal{T}, \mathcal{A})$$

$$\mathcal{T} = \boxed{\{\mathtt{C} \sqsubseteq \mathtt{D}\}}$$

$$\begin{aligned} \mathcal{A} = \{&\mathtt{a} : \mathtt{C}, \\ &\mathtt{a} : \mathtt{C} \sqcup \mathtt{D}, \\ &\mathtt{a} : \neg\mathtt{D}\} \end{aligned}$$



$I_1 :$ $\mathtt{a}^{I_1} \in \mathtt{C}^{I_1}, \mathtt{a}^{I_1} \in \mathtt{D}^{I_1}$

$\boxed{I_2 : \quad \mathtt{a}^{I_2} \in \mathtt{C}^{I_2}, \mathtt{a}^{I_2} \notin \mathtt{D}^{I_2}}$

$I_3 :$ $\mathtt{a}^{I_3} \notin \mathtt{C}^{I_3}, \mathtt{a}^{I_3} \in \mathtt{D}^{I_3}$

$I_4 :$ $\mathtt{a}^{I_4} \notin \mathtt{C}^{I_4}, \mathtt{a}^{I_4} \notin \mathtt{D}^{I_4}$

Probabilistic KB

$$\mathcal{K} = (\mathcal{T}, \mathcal{A})$$

$$\mathcal{T} = \{0.75 :: \mathtt{C} \sqsubseteq \mathtt{D}\}$$

$$\begin{aligned}\mathcal{A} = \{&0.5 :: \mathtt{a} : \mathtt{C},\\ &0.5 :: \mathtt{a} : \mathtt{C} \sqcup \mathtt{D},\\ &0.75 :: \mathtt{a} : \neg\mathtt{D}\}\end{aligned}$$

$$\begin{array}{c}
\begin{array}{cccc}
\mathtt{a}^{I_1} \in \mathtt{C}^{I_1}, \mathtt{a}^{I_1} \in \mathtt{D}^{I_1} & \mathtt{a}^{I_2} \in \mathtt{C}^{I_2}, \mathtt{a}^{I_2} \notin \mathtt{D}^{I_2} & \mathtt{a}^{I_3} \notin \mathtt{C}^{I_3}, \mathtt{a}^{I_3} \in \mathtt{D}^{I_3} & \mathtt{a}^{I_4} \notin \mathtt{C}^{I_4}, \mathtt{a}^{I_4} \notin \mathtt{D}^{I_4}
\end{array}\\
\begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}
\end{array}
\cdot
\begin{bmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \\ \pi_4 \end{bmatrix}
=
\begin{bmatrix} 0.75 \\ 0.5 \\ 0.5 \\ 0.75 \\ 1 \end{bmatrix}$$

Probabilistic KB

$$
\begin{aligned}
\mathcal{K} &= (\mathcal{T}, \mathcal{A}) \\
\mathcal{T} &= \{0.75 :: \mathtt{C} \sqsubseteq \mathtt{D}\} \\
\mathcal{A} &= \{0.5 :: \mathtt{a} : \mathtt{C}, \\
&\qquad 0.5 :: \mathtt{a} : \mathtt{C} \sqcup \mathtt{D}, \\
&\qquad 0.75 :: \mathtt{a} : \neg \mathtt{D}\}
\end{aligned}
$$

$$
\begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \\ \pi_4 \end{bmatrix} = \begin{bmatrix} 0.75 \\ 0.5 \\ 0.5 \\ 0.75 \\ 1 \end{bmatrix}
$$

Probabilistic KB

$$\begin{bmatrix}\pi_1\\ \pi_2\\ \pi_3\\ \pi_4\end{bmatrix} = \begin{bmatrix}0.25\\ 0.25\\ 0\\ 0.5\end{bmatrix}$$

$$\mathcal{K} = (\mathcal{T}, \mathcal{A})$$

$$\mathcal{T} = \{0.75 :: \mathtt{C} \sqsubseteq \mathtt{D}\}$$

$$\begin{aligned}\mathcal{A} = \{&0.5 :: \mathtt{a} : \mathtt{C},\\ &0.5 :: \mathtt{a} : \mathtt{C} \sqcup \mathtt{D},\\ &0.75 :: \mathtt{a} : \neg\mathtt{D}\}\end{aligned}$$

$$\begin{bmatrix}1 & 0 & 1 & 1\\ 1 & 1 & 0 & 0\\ 1 & 1 & 1 & 0\\ 0 & 1 & 0 & 1\\ 1 & 1 & 1 & 1\end{bmatrix} \cdot \begin{bmatrix}\pi_1\\ \pi_2\\ \pi_3\\ \pi_4\end{bmatrix} = \begin{bmatrix}0.75\\ 0.5\\ 0.5\\ 0.75\\ 1\end{bmatrix}$$

Probabilistic KB

$$\begin{bmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \\ \pi_4 \end{bmatrix} = \boxed{\begin{bmatrix} 0.25 \\ 0.25 \\ 0 \\ 0.5 \end{bmatrix}}$$

$$\mathcal{K} = (\mathcal{T}, \mathcal{A})$$

$$\mathcal{T} = \{0.75 :: \mathtt{C} \sqsubseteq \mathtt{D}\}$$

$$\begin{aligned} \mathcal{A} = \{&0.5 :: \mathtt{a} : \mathtt{C}, \\ &0.5 :: \mathtt{a} : \mathtt{C} \sqcup \mathtt{D}, \\ &0.75 :: \mathtt{a} : \neg\mathtt{D}\} \end{aligned}$$

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \\ \pi_4 \end{bmatrix} = \begin{bmatrix} 0.75 \\ 0.5 \\ 0.5 \\ 0.75 \\ 1 \end{bmatrix}$$

Probabilistic KB

$$\mathcal{K} = (\mathcal{T}, \mathcal{A})$$

$$\mathcal{T} = \{0.75 :: \mathtt{C} \sqsubseteq \mathtt{D}\}$$

$$\begin{aligned}\mathcal{A} = \{&0.75 :: \mathtt{a} : \mathtt{C},\\ &0.75 :: \mathtt{a} : \mathtt{C} \sqcup \mathtt{D},\\ &0.75 :: \mathtt{a} : \neg\mathtt{D}\}\end{aligned}$$

Probabilistic KB

$$\mathcal{K} = (\mathcal{T}, \mathcal{A})$$

$$\mathcal{T} = \{0.75 :: \mathtt{C} \sqsubseteq \mathtt{D}\}$$

$$\mathcal{A} = \{0.75 :: \mathtt{a} : \mathtt{C},$$

$$0.75 :: \mathtt{a} : \mathtt{C} \sqcup \mathtt{D},$$

$$0.75 :: \mathtt{a} : \neg\mathtt{D}\}$$

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \\ \pi_4 \end{bmatrix} = \begin{bmatrix} 0.75 \\ 0.75 \\ 0.75 \\ 0.75 \\ 1 \end{bmatrix}$$

Probabilistic KB

NO SOLUTION

$$\mathcal{K} = (\mathcal{T}, \mathcal{A})$$

$$\mathcal{T} = \{0.75 :: \mathtt{C} \sqsubseteq \mathtt{D}\}$$

$$\mathcal{A} = \{0.75 :: \mathtt{a} : \mathtt{C},$$

$$0.75 :: \mathtt{a} : \mathtt{C} \sqcup \mathtt{D},$$

$$0.75 :: \mathtt{a} : \neg\mathtt{D}\}$$

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \cdot \boxed{\begin{bmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \\ \pi_4 \end{bmatrix}} = \begin{bmatrix} 0.75 \\ 0.75 \\ 0.75 \\ 0.75 \\ 1 \end{bmatrix}$$

Motivation

- What is the source of a probability value?

Motivation

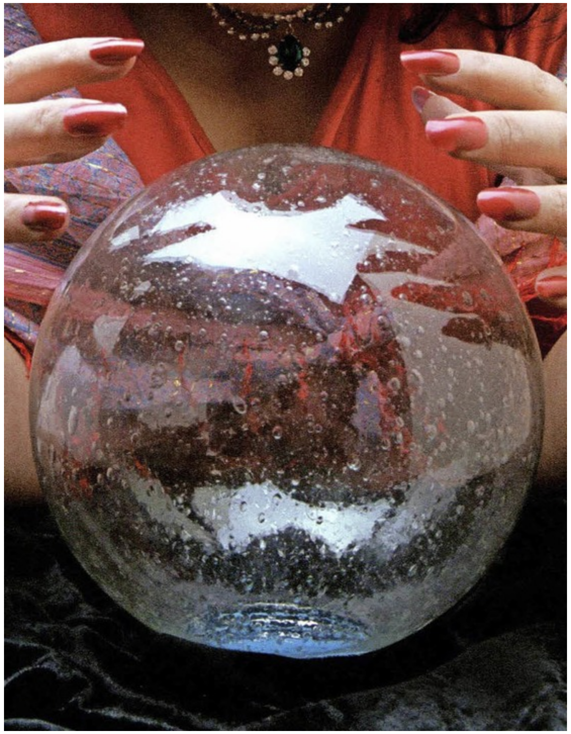
- What is the source of a probability value?
  - Symmety Aspects

Motivation

- What is the source of a probability value?
  - Symmety Aspects
  - Statistical Tests

Motivation

- What is the source of a probability value?
  - Symmety Aspects
  - Statistical Tests
  - Opinion / Estimation



Motivation

- What is the source of a probability value?
  - Symmety Aspects
  - Statistical Tests
  - Opinion / Estimation

Clarification

- Classical Probabilistic DL

  75% :: Axiom

  => Axiom is true in 3 of 4 possible worlds

Clarification

- Axiombased Probabilistic DL

  75% :: Axiom

  => 3 of 4 such axioms are true within the same knowledge base

Confidence of a Possible World

$$\sigma(I) = \frac{1}{|\mathcal{K}|} \cdot \left(\sum_{\substack{p::\phi \in \mathcal{K} \\ I \models \phi}} (1 - p) - \sum_{\substack{p::\phi \in \mathcal{K} \\ I \not\models \phi}} p \right)$$

Calculation

$$\mathcal{K} = (\mathcal{T}, \mathcal{A})$$

$$\mathcal{T} = \{0.75 :: \mathtt{C} \sqsubseteq \mathtt{D}\}$$

$$\begin{aligned}\mathcal{A} = \{&0.75 :: \mathtt{a} : \mathtt{C},\\ &0.75 :: \mathtt{a} : \mathtt{C} \sqcup \mathtt{D},\\ &0.75 :: \mathtt{a} : \neg \mathtt{D}\}\end{aligned}$$

$$\mathtt{a}^{I_1} \in \mathtt{C}^{I_1}, \mathtt{a}^{I_1} \in \mathtt{D}^{I_1} \qquad \sigma(I_1) = \frac{1}{4}(0.25 + 0.25 + 0.25 - 0.75) =$$

$$\mathtt{a}^{I_2} \in \mathtt{C}^{I_2}, \mathtt{a}^{I_2} \notin \mathtt{D}^{I_2} \qquad \sigma(I_2) = \frac{1}{4}(-0.75 + 0.25 + 0.25 + 0.25)$$

$$\mathtt{a}^{I_3} \notin \mathtt{C}^{I_3}, \mathtt{a}^{I_3} \in \mathtt{D}^{I_3} \qquad \sigma(I_3) = \frac{1}{4}(0.25 - 0.75 + 0.25 - 0.75) =$$

$$\mathtt{a}^{I_4} \notin \mathtt{C}^{I_4}, \mathtt{a}^{I_4} \notin \mathtt{D}^{I_4} \qquad \sigma(I_4) = \frac{1}{4}(0.25 - 0.75 - 0.75 + 0.25) =$$

Calculation

$$\mathcal{K} = (\mathcal{T}, \mathcal{A})$$

$$\mathcal{T} = \boxed{\{0.75 :: \mathtt{C} \sqsubseteq \mathtt{D}\}}$$

$$\begin{aligned} \mathcal{A} = \{&0.75 :: \mathtt{a} : \mathtt{C}, \\ &0.75 :: \mathtt{a} : \mathtt{C} \sqcup \mathtt{D}, \\ &0.75 :: \mathtt{a} : \neg \mathtt{D}\} \end{aligned}$$

$$\boxed{\mathtt{a}^{I_1} \in \mathtt{C}^{I_1}, \mathtt{a}^{I_1} \in \mathtt{D}^{I_1}} \qquad \sigma(I_1) = \frac{1}{4}(\boxed{0.25} + 0.25 + 0.25 - 0.75) =$$

$$\mathtt{a}^{I_2} \in \mathtt{C}^{I_2}, \mathtt{a}^{I_2} \notin \mathtt{D}^{I_2} \qquad \sigma(I_2) = \frac{1}{4}(-0.75 + 0.25 + 0.25 + 0.25)$$

$$\mathtt{a}^{I_3} \notin \mathtt{C}^{I_3}, \mathtt{a}^{I_3} \in \mathtt{D}^{I_3} \qquad \sigma(I_3) = \frac{1}{4}(0.25 - 0.75 + 0.25 - 0.75) =$$

$$\mathtt{a}^{I_4} \notin \mathtt{C}^{I_4}, \mathtt{a}^{I_4} \notin \mathtt{D}^{I_4} \qquad \sigma(I_4) = \frac{1}{4}(0.25 - 0.75 - 0.75 + 0.25) =$$

Calculation

$$\mathcal{K} = (\mathcal{T}, \mathcal{A})$$

$$\mathcal{T} = \{0.75 :: \mathtt{C} \sqsubseteq \mathtt{D}\}$$

$$\begin{aligned} \mathcal{A} = &\boxed{\{0.75 :: \mathtt{a} : \mathtt{C},} \\ &0.75 :: \mathtt{a} : \mathtt{C} \sqcup \mathtt{D}, \\ &0.75 :: \mathtt{a} : \neg \mathtt{D}\} \end{aligned}$$

$$\boxed{\mathtt{a}^{I_1} \in \mathtt{C}^{I_1}, \mathtt{a}^{I_1} \in \mathtt{D}^{I_1}} \qquad \sigma(I_1) = \frac{1}{4}(0.25 \boxed{+0.25} +0.25 - 0.75) =$$

$$\mathtt{a}^{I_2} \in \mathtt{C}^{I_2}, \mathtt{a}^{I_2} \notin \mathtt{D}^{I_2} \qquad \sigma(I_2) = \frac{1}{4}(-0.75 + 0.25 + 0.25 + 0.25)$$

$$\mathtt{a}^{I_3} \notin \mathtt{C}^{I_3}, \mathtt{a}^{I_3} \in \mathtt{D}^{I_3} \qquad \sigma(I_3) = \frac{1}{4}(0.25 - 0.75 + 0.25 - 0.75) =$$

$$\mathtt{a}^{I_4} \notin \mathtt{C}^{I_4}, \mathtt{a}^{I_4} \notin \mathtt{D}^{I_4} \qquad \sigma(I_4) = \frac{1}{4}(0.25 - 0.75 - 0.75 + 0.25) =$$

Calculation

$$\mathcal{K} = (\mathcal{T}, \mathcal{A})$$

$$\mathcal{T} = \{0.75 :: \mathtt{C} \sqsubseteq \mathtt{D}\}$$

$$\begin{aligned} \mathcal{A} = \{&0.75 :: \mathtt{a} : \mathtt{C}, \\ &\boxed{0.75 :: \mathtt{a} : \mathtt{C} \sqcup \mathtt{D},} \\ &0.75 :: \mathtt{a} : \neg \mathtt{D}\} \end{aligned}$$

$$\boxed{\mathtt{a}^{I_1} \in \mathtt{C}^{I_1}, \mathtt{a}^{I_1} \in \mathtt{D}^{I_1}} \qquad \sigma(I_1) = \frac{1}{4}(0.25 + 0.25 \boxed{+ 0.25} - 0.75) =$$

$$\mathtt{a}^{I_2} \in \mathtt{C}^{I_2}, \mathtt{a}^{I_2} \notin \mathtt{D}^{I_2} \qquad \sigma(I_2) = \frac{1}{4}(-0.75 + 0.25 + 0.25 + 0.25)$$

$$\mathtt{a}^{I_3} \notin \mathtt{C}^{I_3}, \mathtt{a}^{I_3} \in \mathtt{D}^{I_3} \qquad \sigma(I_3) = \frac{1}{4}(0.25 - 0.75 + 0.25 - 0.75) =$$

$$\mathtt{a}^{I_4} \notin \mathtt{C}^{I_4}, \mathtt{a}^{I_4} \notin \mathtt{D}^{I_4} \qquad \sigma(I_4) = \frac{1}{4}(0.25 - 0.75 - 0.75 + 0.25) =$$

Calculation

$$\mathcal{K} = (\mathcal{T}, \mathcal{A})$$

$$\mathcal{T} = \{0.75 :: \mathtt{C} \sqsubseteq \mathtt{D}\}$$

$$\mathcal{A} = \{0.75 :: \mathtt{a} : \mathtt{C},$$

$$0.75 :: \mathtt{a} : \mathtt{C} \sqcup \mathtt{D},$$

$$\boxed{0.75 :: \mathtt{a} : \neg \mathtt{D}\}}$$

$$\boxed{\mathtt{a}^{I_1} \in \mathtt{C}^{I_1}, \mathtt{a}^{I_1} \in \mathtt{D}^{I_1}} \qquad \sigma(I_1) = \frac{1}{4}(0.25 + 0.25 + 0.25 \boxed{- 0.75)} =$$

$$\mathtt{a}^{I_2} \in \mathtt{C}^{I_2}, \mathtt{a}^{I_2} \notin \mathtt{D}^{I_2} \qquad \sigma(I_2) = \frac{1}{4}(-0.75 + 0.25 + 0.25 + 0.25)$$

$$\mathtt{a}^{I_3} \notin \mathtt{C}^{I_3}, \mathtt{a}^{I_3} \in \mathtt{D}^{I_3} \qquad \sigma(I_3) = \frac{1}{4}(0.25 - 0.75 + 0.25 - 0.75) =$$

$$\mathtt{a}^{I_4} \notin \mathtt{C}^{I_4}, \mathtt{a}^{I_4} \notin \mathtt{D}^{I_4} \qquad \sigma(I_4) = \frac{1}{4}(0.25 - 0.75 - 0.75 + 0.25) =$$

Interpretation

- The Confidence Value is between -1 and 1
- If the maximum confidence value is close to 0, it is likely to be the correct world

Algorithm

- Find the possible world with the highest confidence value

Algorithm

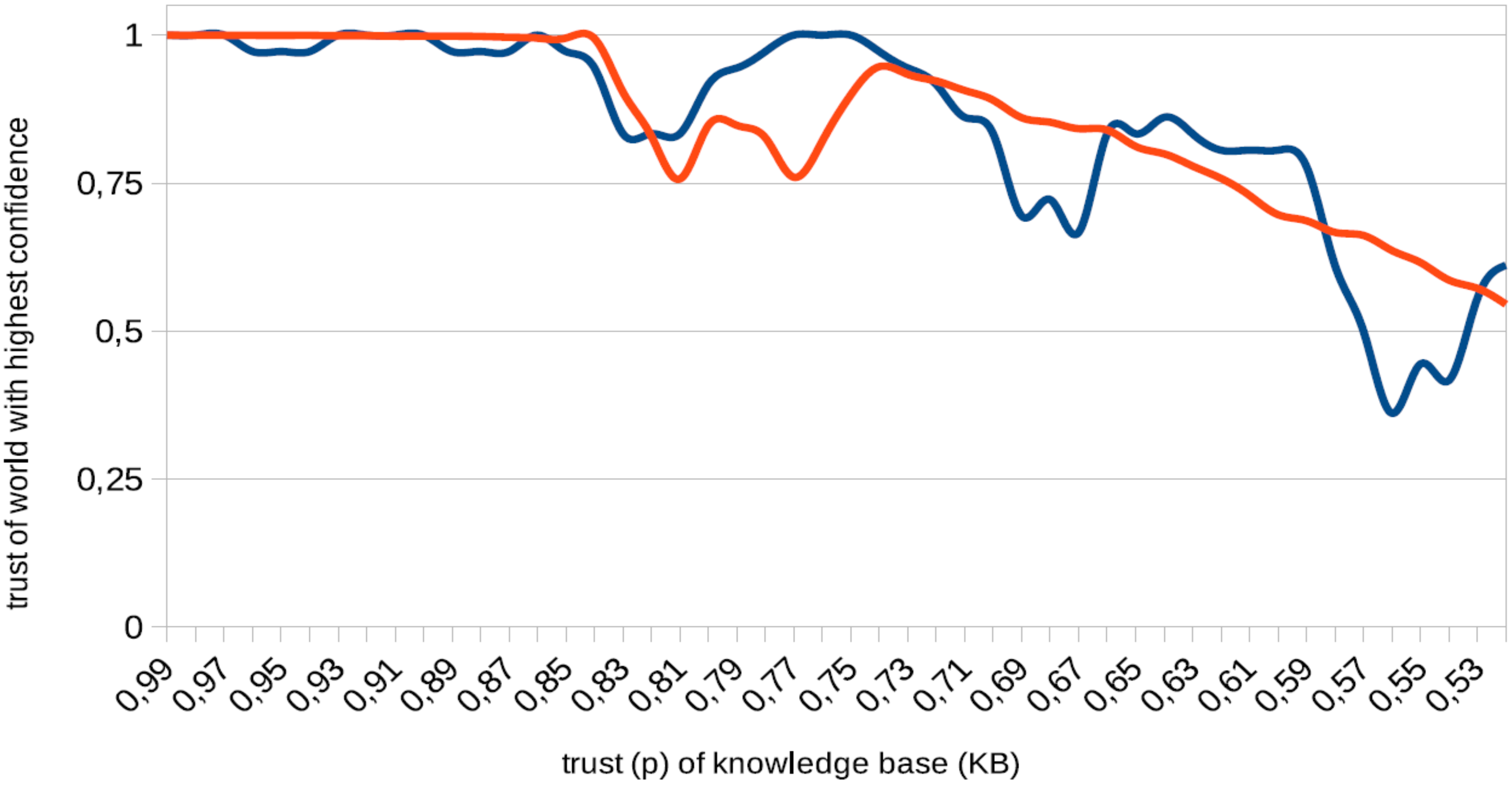
- Find the possible world with the highest confidence value
  - Optimization Problem

Application

- Improving Trust of given Knowledge Base with a certain trust level

- Input: KB (90% of Axioms are true)
  => Perform algorithm
  Output: KB with highest confidence

Test Results



Thank You for Your Attention!

Martin Unold & Christophe Cruz