Modeling Cardinal Direction Calculus in a Fuzzy Description Logic

A Comparison of Different Approaches

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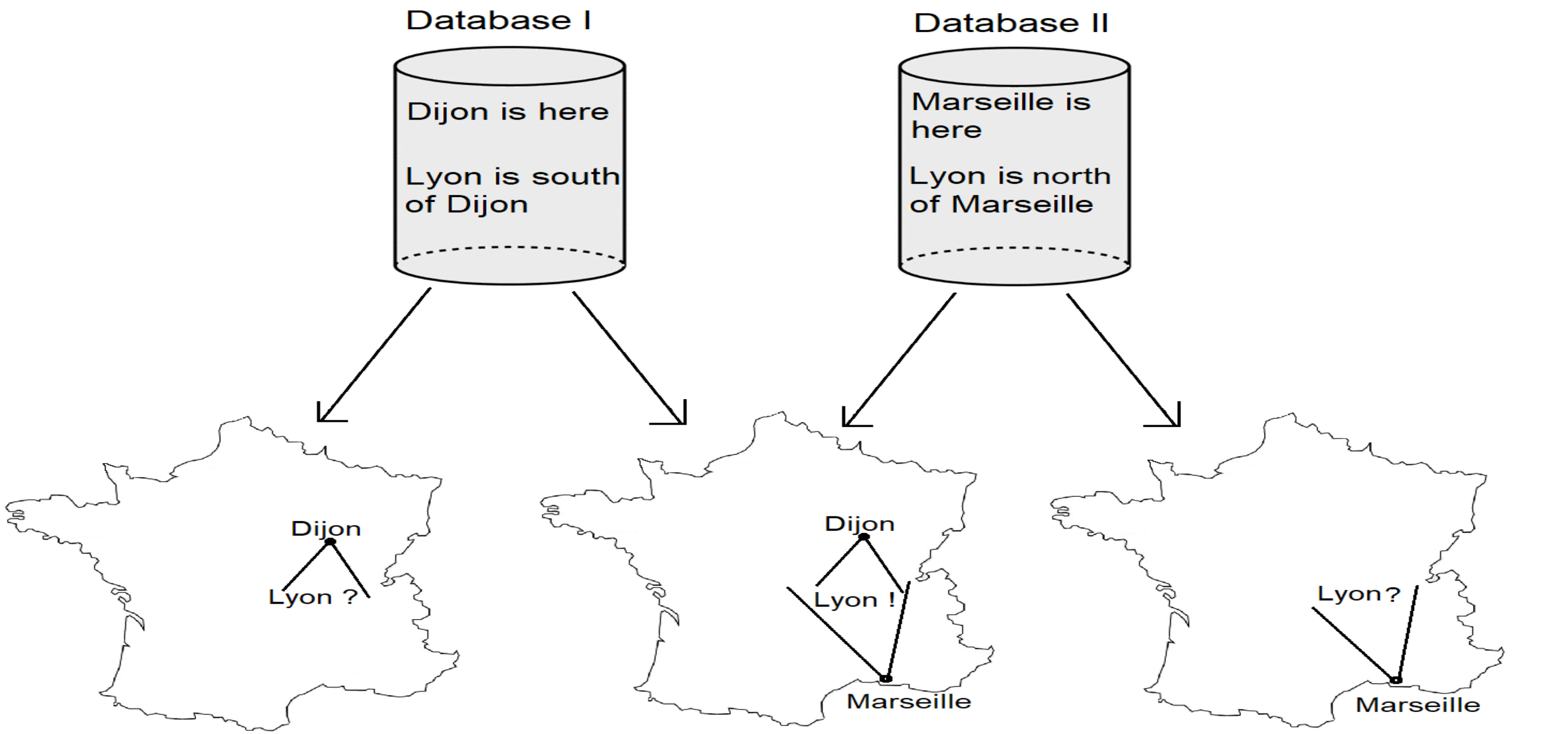
Outline

- Motivation (crisp)
- Motivation (fuzzy)
- Inference Rules
- Test Results

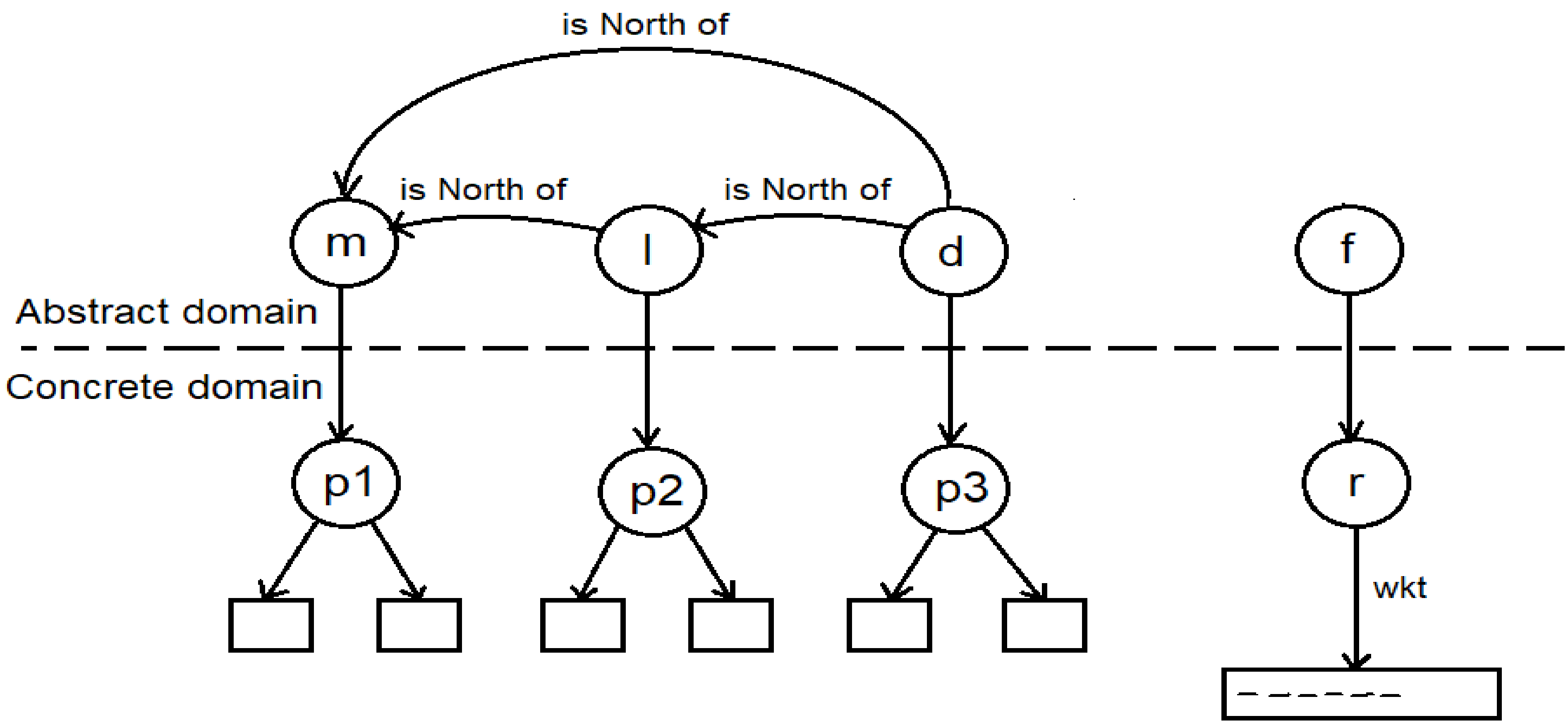
Typical Situation



- (lyon,marseille): northOf
- (lyon,france): within
- (marseille,avignon): closeTo

- (lyon,point2): isHere
- (france,region1): isHere

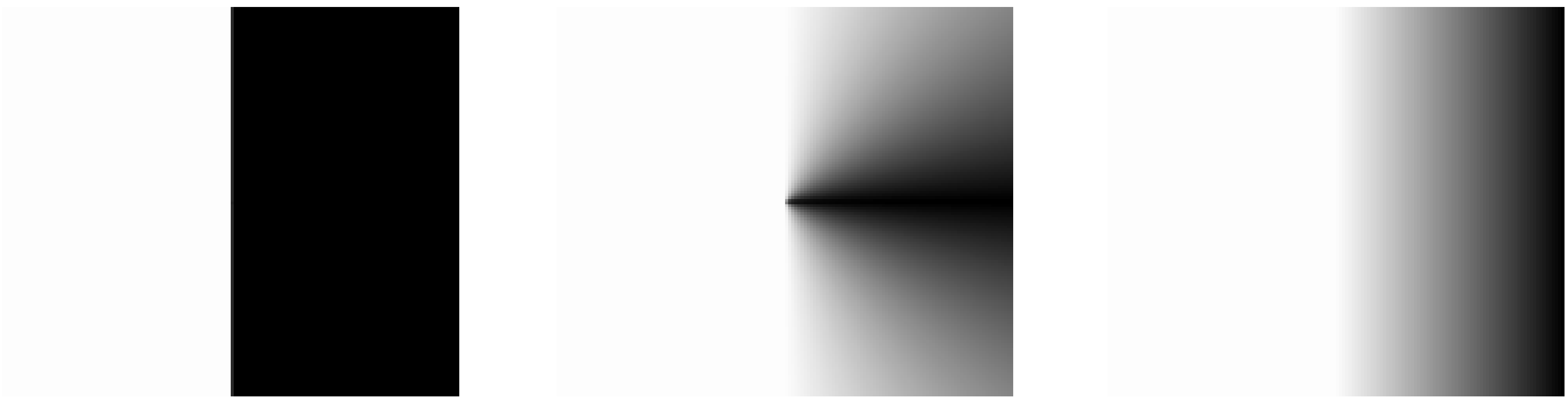
Model in DL


is North of
is North of
is North of
m
l
d
f
Abstract domain
Concrete domain
p1
p2
p3
r
wkt

- <(lyon,marseille): northOf, 70%>

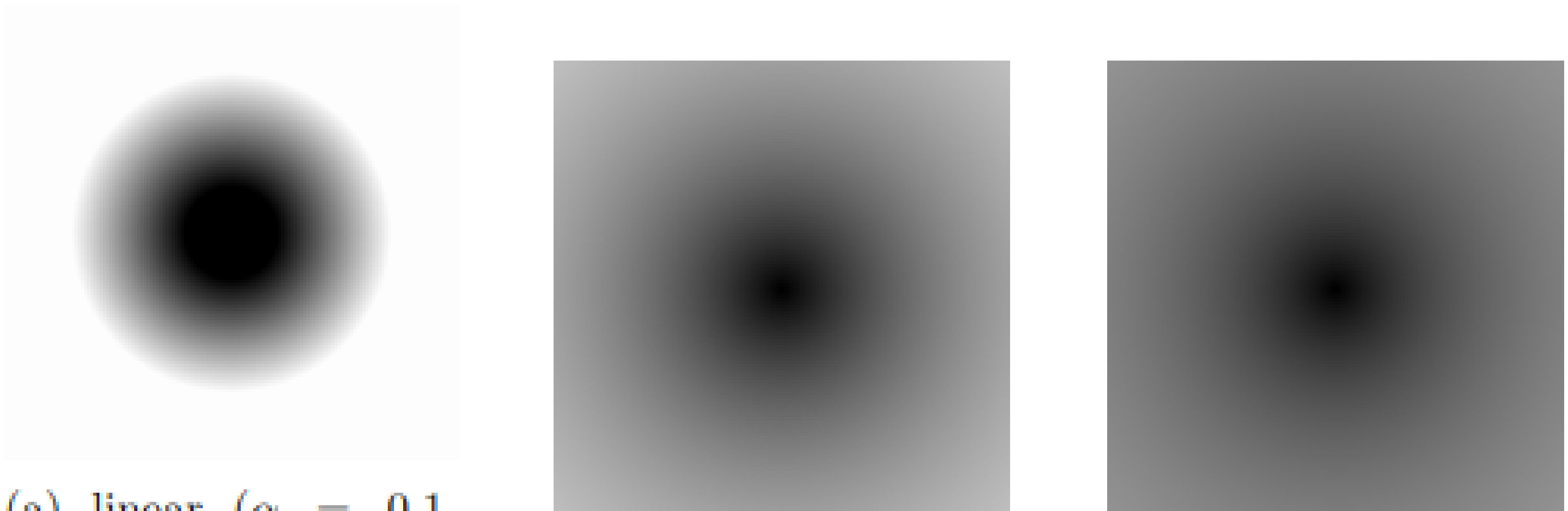
- <(lyon,marseille): northOf, 70%>
- This is NOT a probability !!!

What is East?



(a) crisp (b) cone (c) projection

What is Close?



(a) linear ($\alpha = 0.1$, $\beta = 0.25$)

(b) exponential

(c) fraction

$$\mu(d) = \begin{cases} 0 & \text{if } d \geq \alpha + \beta \\ \frac{\alpha+\beta-d}{\beta} & \text{if } \alpha < d < \alpha + \beta \\ 1 & \text{if } d \leq \alpha \end{cases}$$

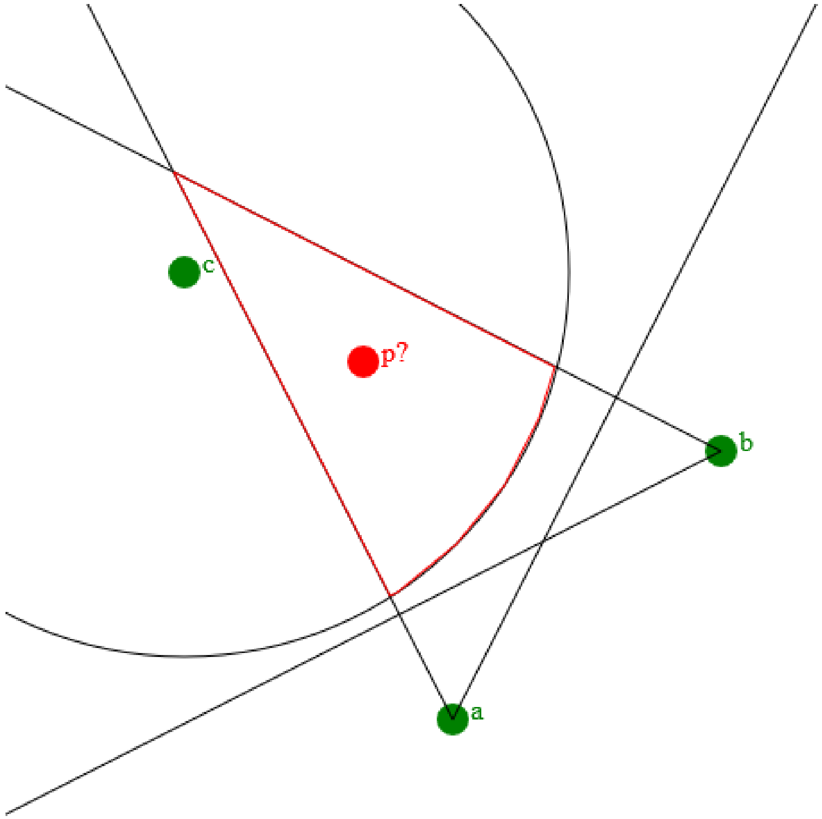
$$\mu(d) = e^{-d}$$

$$\mu(d) = \frac{1}{1+d}$$

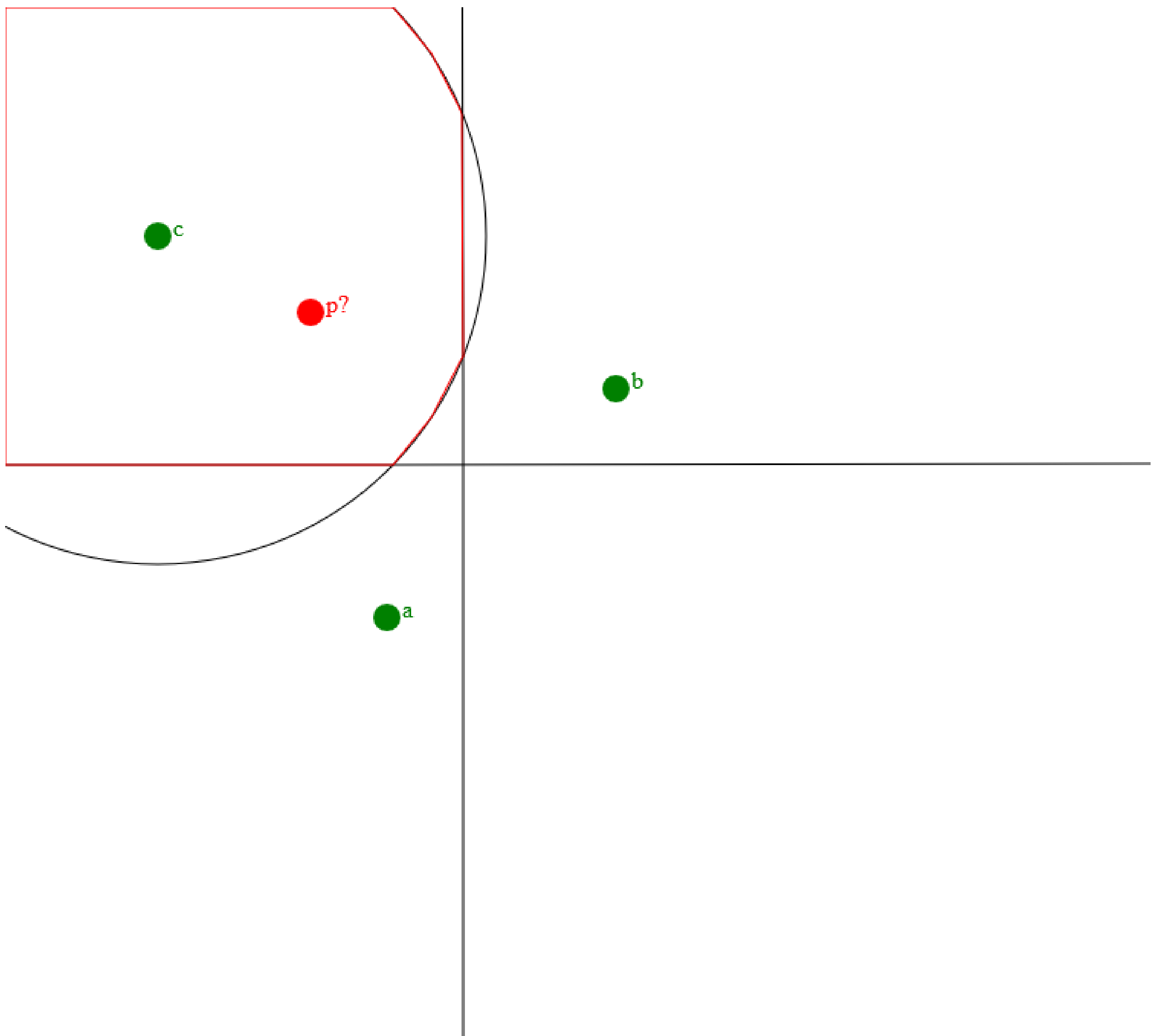
Where is Point P?

- P is northOf A (60%)
- P is westOf B (70%)
- P is closeTo C (80%)

Cone Model



Projection Model



Inference Rules

- A is east of B
- B is east of C

=> A is east of C ?

Inference Rules

- A is east of B (x %)
- B is east of C (y %)

=> A is east of C (x+y? min(x,y)? ...)

Fuzzy Connectives

	Lukasiewicz Logic	Product Logic	Goedel Logic
$\ominus\phi =$	$1-\phi$	$\begin{cases} 1 & \text{if } \phi = 0 \\ 0 & \text{if } \phi > 0 \end{cases}$	$\begin{cases} 1 & \text{if } \phi = 0 \\ 0 & \text{if } \phi > 0 \end{cases}$
$\phi_1 \oplus \phi_2 =$	$\min(\phi_1 + \phi_2, 1)$	$\phi_1 + \phi_2 - \phi_1 \cdot \phi_2$	$\max(\phi_1, \phi_2)$
$\phi_1 \otimes \phi_2 =$	$\max(\phi_1 + \phi_2 - 1, 0)$	$\phi_1 \cdot \phi_2$	$\min(\phi_1, \phi_2)$
$\phi_1 \rhd \phi_2 =$	$\min(1 - \phi_1 + \phi_2, 1)$	$\min\left(1, \frac{\phi_2}{\phi_1}\right)$	$\begin{cases} 1 & \text{if } \phi_1 \leq \phi_2 \\ \phi_2 & \text{if } \phi_1 > \phi_2 \end{cases}$

Fuzzy Connectives

	Lukasiewicz Logic	Product Logic	Goedel Logic
$\ominus\phi =$	$1 - \phi$	$\begin{cases} 1 & \text{if } \phi = 0 \\ 0 & \text{if } \phi > 0 \end{cases}$	$\begin{cases} 1 & \text{if } \phi = 0 \\ 0 & \text{if } \phi > 0 \end{cases}$
$\phi_1 \oplus \phi_2 =$	$\min(\phi_1 + \phi_2, 1)$	$\phi_1 + \phi_2 - \phi_1 \cdot \phi_2$	$\max(\phi_1, \phi_2)$
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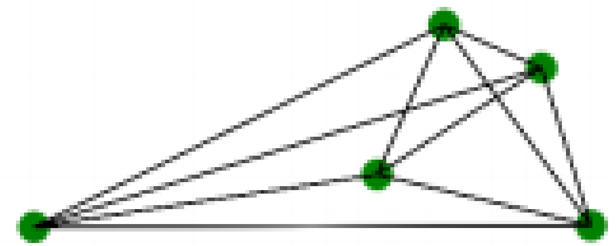
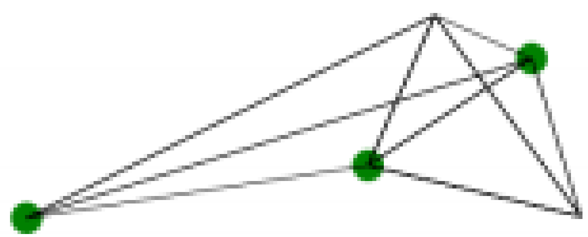
Inference Rules

Axiom	cone-based	projection-based
$\mathtt{E} \circ \mathtt{E} \sqsubseteq \mathtt{E}$	G	G
$\mathtt{E} \sqcap \mathtt{N} \sqsubseteq \mathtt{NE}$	G	G
$\mathtt{E} \circ \mathtt{N} \sqsubseteq \mathtt{NE}$	$\times$	$\times$
$\mathtt{E}^{-} \sqsubseteq \mathtt{W}$	✓	✓

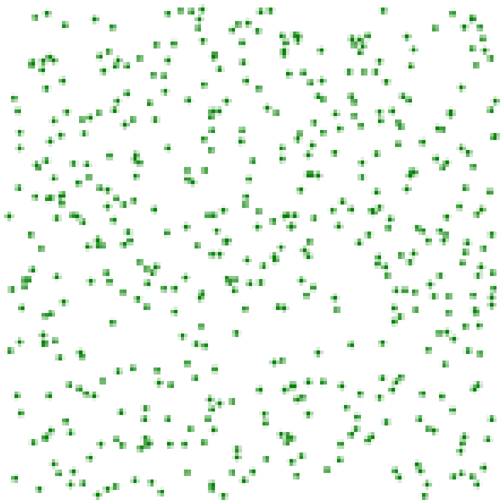
Axiom	linear ($\alpha > 0$)	linear ($\alpha = 0$)	exponential	fractional
$\mathtt{Near} \circ \mathtt{Near} \sqsubseteq \mathtt{Near}$	$\times$	L	P	P
$\mathtt{Near}^{-} \sqsubseteq \mathtt{Near}$	✓	✓	✓	✓



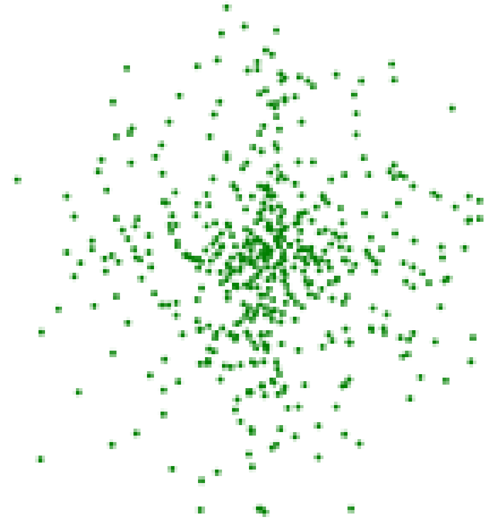
(a) Complete dataset: Black lines state available relative information. Green dots represent absolute information.



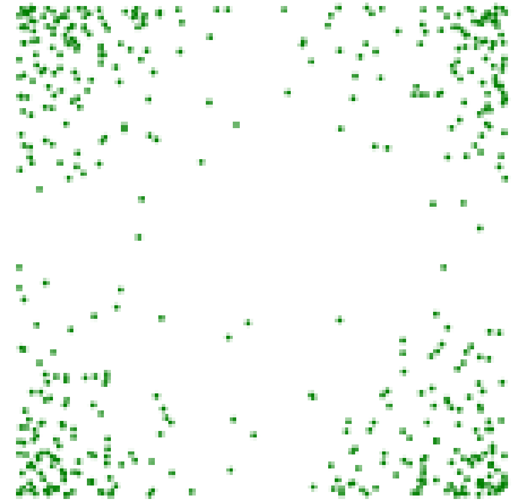
(b) Damaged dataset: Only $\tau = 60\%$ of absolute information is available and $\rho = 60\%$ of relative data is removed.



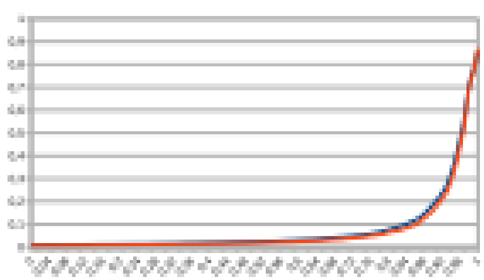
(a) equal distribution: $f(p) = 1$

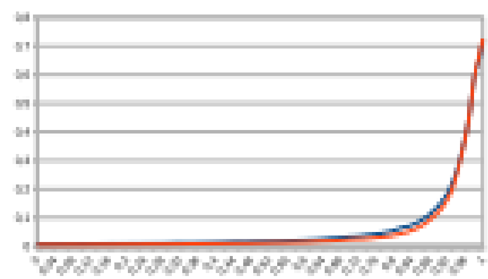


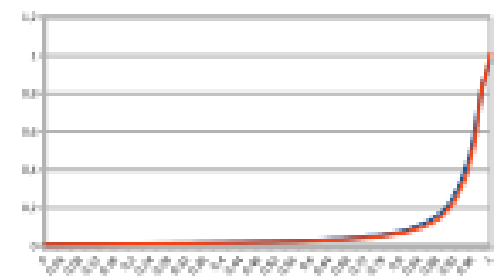
(b) centered distribution: $f(p) = (2 * p_x - 1)^2 + (2 * p_y - 1)^2$



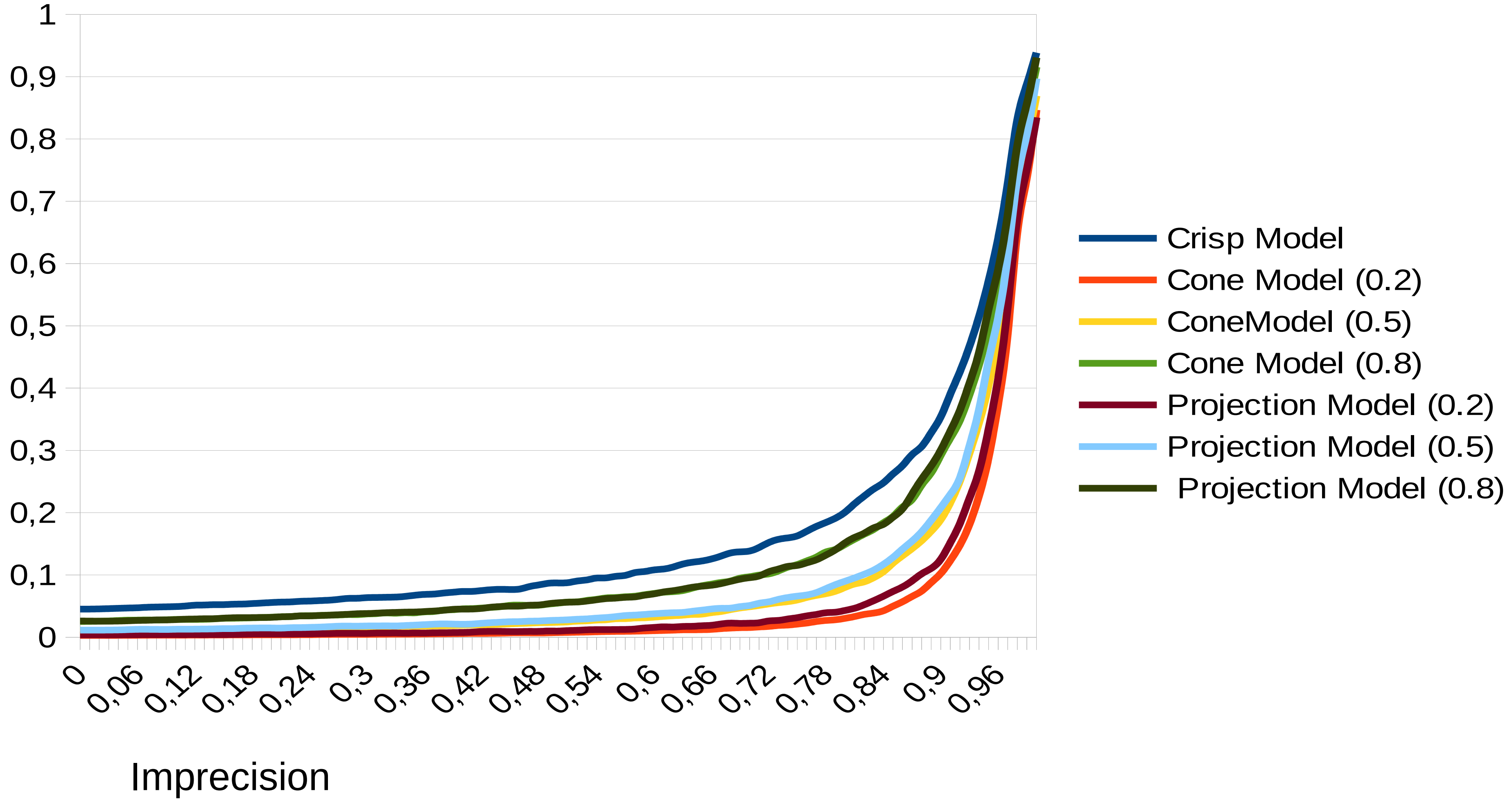
(c) bordered distribution: $f(p) = 2 - (2 * p_x - 1)^2 + (2 * p_y - 1)^2$



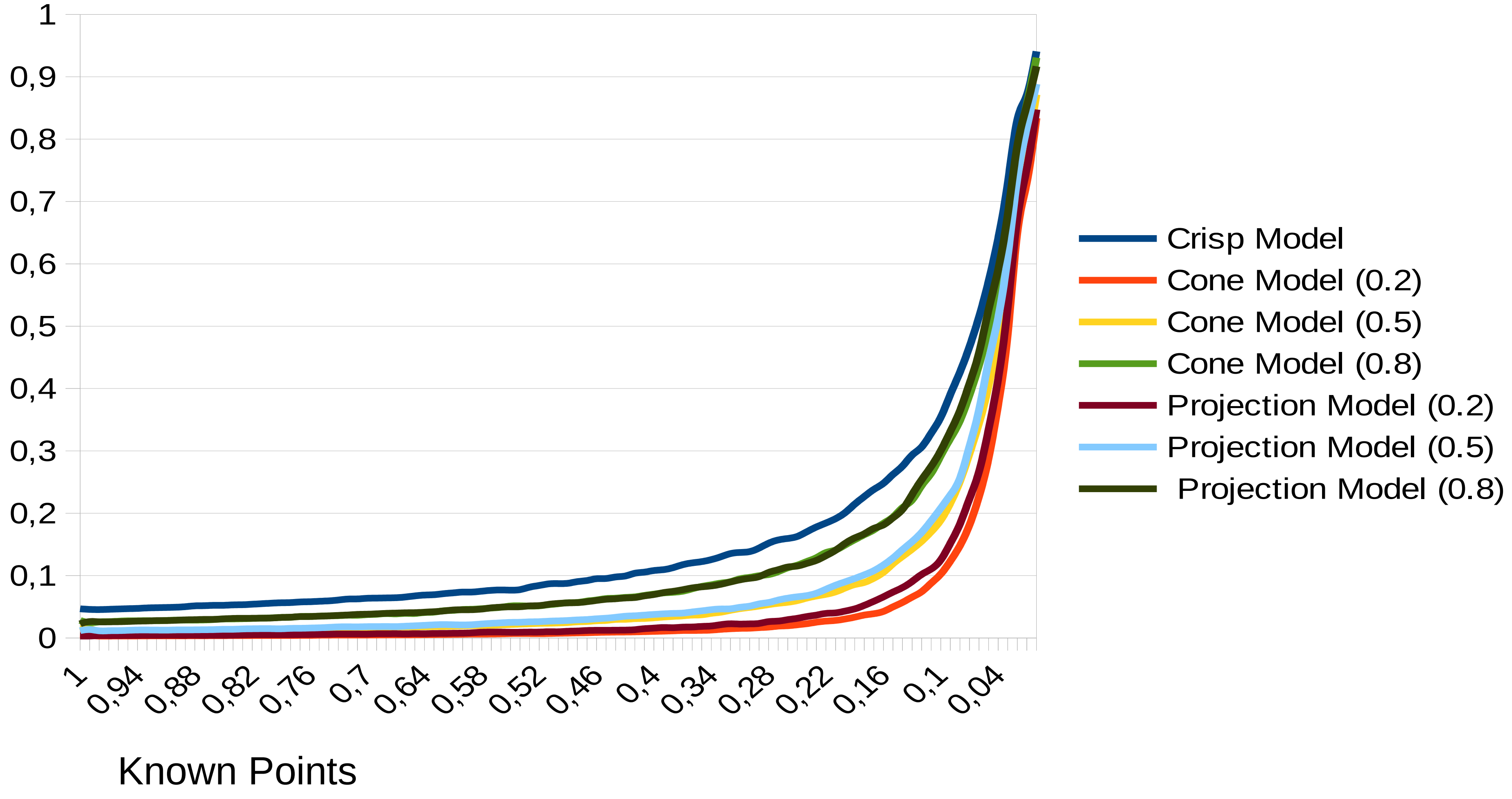




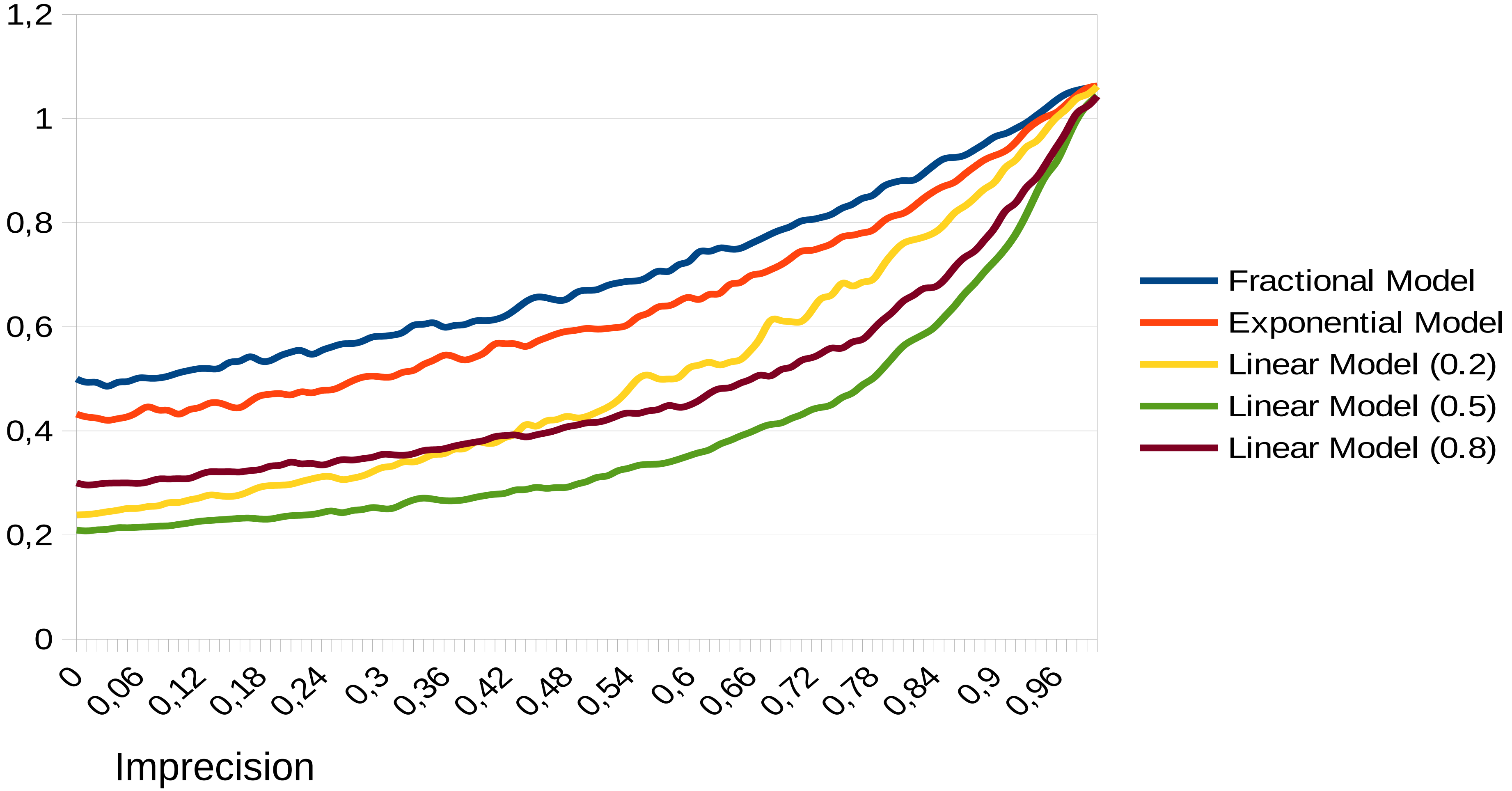
Cardinal Directions



Cardinal Directions

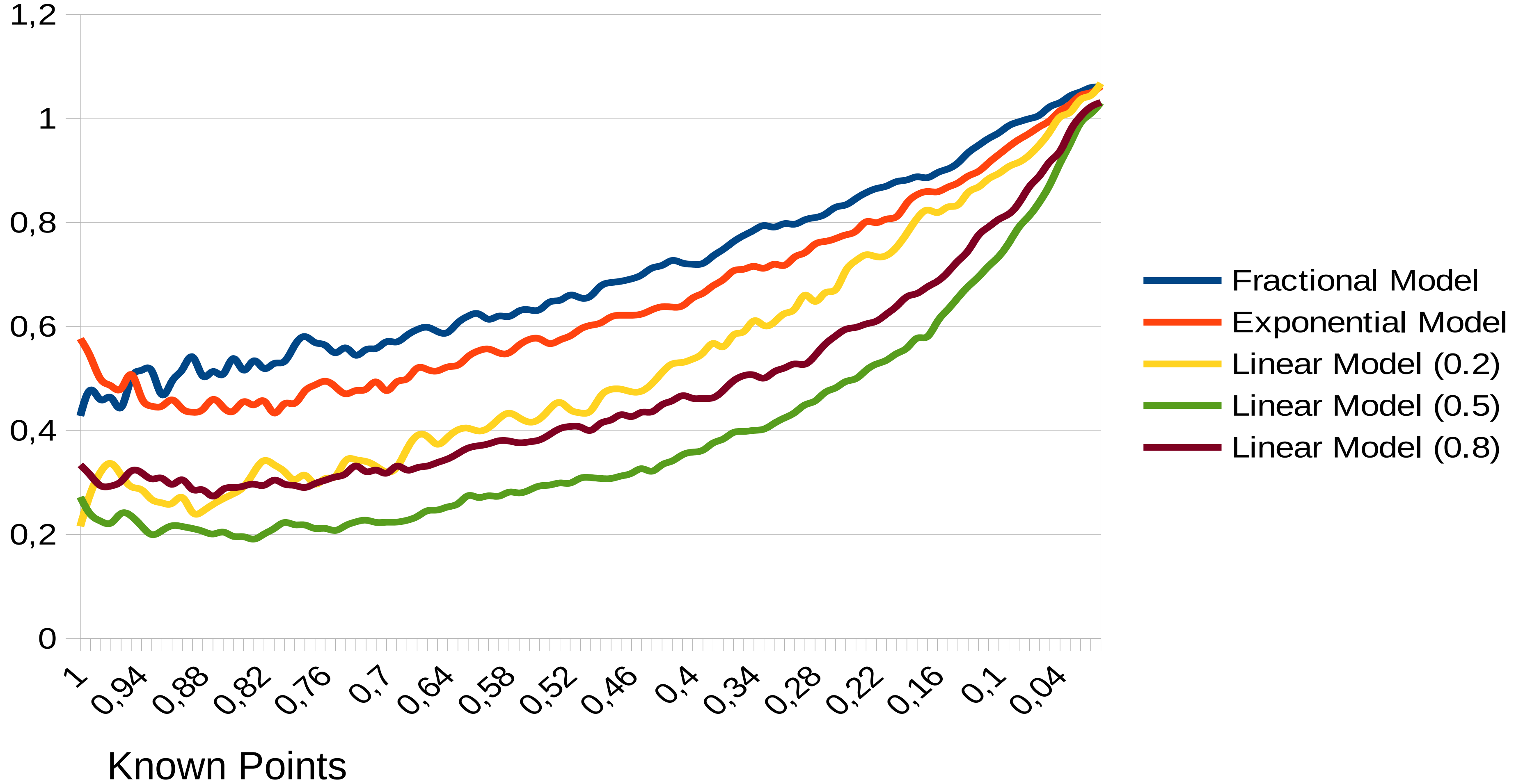


Nearness



1,2
1
0,8
0,6
0,4
0,2
0
0
0,06
0,12
0,18
0,24
0,3
0,36
0,42
0,48
0,54
0,6
0,66
0,72
0,78
0,84
0,9
0,96
Imprecision
Fractional Model
Exponential Model
Linear Model (0.2)
Linear Model (0.5)
Linear Model (0.8)

Nearness



1,2
1
0,8
0,6
0,4
0,2
0
1
0,94
0,88
0,82
0,76
0,7
0,64
0,58
0,52
0,46
0,4
0,34
0,28
0,22
0,16
0,1
0,04
Known Points
Fractional Model
Exponential Model
Linear Model (0.2)
Linear Model (0.5)
Linear Model (0.8)

Thank You for Your Attention!

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